

[Time: 2¹/₂ Hours]

[Marks:75]

Please check whether you have got the right question paper.

- N.B: 1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 Answer following questions.

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- 1) Let f be defined on an interval, and let x_1 and x_2 be points on the interval, then f is said to be decreasing if
 - p) $f(x_1) > f(x_2)$ whenever $x_1 < x_2$
 - q) $f(x_1) > f(x_2)$ whenever $x_1 > x_2$
 - r) $f(x_1) = f(x_2)$ whenever $x_1 < x_2$
 - s) None of these
- 2) If a function f is concave up on (a, b) then which of the following is true on (a, b)
 - p) $f' > 0$
 - q) $f' < 0$
 - r) $f' = 0$
 - s) None of these
- 3) If f is integrable on $[a, b]$ and $f(x) \geq 0 \forall x \in [a, b]$, then
 - p) $\int_a^b f(x) > 0$
 - q) $\int_a^b f(x) \geq 0$
 - r) $\int_a^b f(x) = 0$
 - s) None of these
- 4) A rule that assigns a unique real number $f(x, y)$ to each point (x, y) in some set D in the xy -plane is called
 - p) a function of one variable
 - q) a function of two variable
 - r) a function of three variables
 - s) None of these
- 5) which of the following is true about the function $f(x, y) = 3x^2y^5$?
 - p) Discontinuous at $(0, 0)$
 - q) Discontinuous at $(1, 1)$
 - r) Continuous everywhere
 - s) None of these

b) Fill in the blanks for the following questions:

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- 1) A function f has a relative maximum at x_0 if there is an open interval containing x_0 on which $f(x)$ is ---- $f(x_0)$ for every x in the interval.
- 2) The points on the curve $y=f(x)$ where the rate of change of y with respect to x changes from increasing to decreasing, or vice versa is known as-----
- 3) The integral $\int_0^\pi \sqrt{(1 + \cos x)^2} dx$ is the arc length of $y=-----$ from $x=0$ to $x=\pi$.
- 4) If $f(x, y) = \frac{x-y}{x+y+1}$, the value of $f(y+1, y)$ is given by -----
- 5) The value of $\lim_{(x,y) \rightarrow (0,1)} e^{xy^2} = -----$

(Turn Over)

c) State true or false for the following questions:

- 1) If $f(x)=0$ has a root, then Newtons Method starting at $x=x_1$ will approximate the root nearest x_1 .
- 2) The order of the differential equation $\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$ is one.
- 3) If $f(x,y) \rightarrow L$ as (x,y) approaches $(0,0)$ along the x-axis, and if $f(x,y) \rightarrow L$ as (x,y) approaches $(0,0)$ along the y-axis then $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = L$.
- 4) If a function f is continuous at every point in an open set D , then f is continuous on D .
- 5) A function f of two variables is said to have a relative maximum at a point (x_0, y_0) if there is a disk centered at (x_0, y_0) such that $f(x_0, y_0) \leq f(x, y)$ for all points (x, y) that lie inside the disk.

Q.2 Answer any THREE of the following questions:

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- a) Find the intervals on which $f(x) = x^2 - 3x + 8$ is increasing and the intervals on which it is decreasing.
- b) Use first and second derivative tests to show that $f(x) = 3x^2 - 6x + 1$ has a relative minimum at $x=1$.
- c) Sketch the graph of the equation $y = x^3 - 3x + 2$ and identify the locations of the intercepts (draw the graph on the answer sheet itself).
- d) Find the absolute maximum and minimum values of $f(x) = (x-2)^2$ in $[1, 4]$.
- e) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?
- f) The equation $x^3 - 2x - 2 = 0$ has one real solution. Approximate it by Newtons Method.

Q.3 Answer any THREE of the following questions:

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- a) Find the area under the curve $y = x^3$ over the interval $[2, 3]$.
- b) Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$ and bounded on the sides by the lines $x = 0$ and $x = 2$.
- c) Find the approximate value of $\int_{1/2}^2 \frac{1}{x} dx$ using Simpson's rule with $n=10$.
- d) Solve differential equation $\frac{dy}{dx} = \frac{y}{x}$.
- e) Use Euler's Method with a step size of 0.25 to find approximate solution of the initial-value problem $\frac{dy}{dx} = x - y^2, y(x) = 1$ over $0 \leq x \leq 1$.
- f) Solve the differential equation $\frac{dy}{dx} + 3y = e^{-2x}$ by the method of integrating factors.

Q.4 Answer any THREE of the following questions:

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- a) If $f(x, y) = \frac{xy}{x^2 + y^2}$, find the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ 1) along x-axis and 2) along the line $y=x$.
- b) Evaluate $\lim_{(x,y) \rightarrow (0,0)} \sqrt{x^2 + y^2} \cdot \log(x^2 + y^2)$, by converting to polar coordinates.
- c) Find $f_x(x, y)$ and $f_y(x, y)$ for $f(x, y) = 2x^3y^2 + 2y + 4x$, and use those partial derivatives to compute $f_x(1, 3)$ and $f_y(1, 3)$.
- d) Find the directional derivative of $f(x, y) = e^{xy}$ at $(2, 0)$ in the direction of unit vector that makes an angle of $\frac{\pi}{3}$ with the positive x-axis.
- e) Find an equation of the tangent plane to the surface $x^2 + 4y^2 + z^2 = 18$ at the point $(1, 2, 1)$. Also find the parametric equation of the line that is normal to the surface at the point $(1, 2, 1)$.
- f) Find all relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

(Turn Over)

Q.5 Answer any THREE of the following questions:

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- Find the absolute maximum and minimum values of $f(x) = \frac{x-2}{x+2}$ on $[-1,5]$.
- Show that for any constants A and B, the function $y = Ae^{2x} + Be^{-4x}$ satisfies the equation $y'' + 2y - 8y = 0$.
- Find the area of the region under the curve $y = x^2 + 1$ and over the interval $[0,3]$.
- Solve differential equation $\frac{dy}{dx} + 2xy = x$.
- Determine whether the following limit exists. If so, find its value. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 16y^4}{x^2 + 4y^2}$.