Q.P. Code : 781300

(2½ Hours) [Total Marks: 75]

N.B. (1) All questions are compulsory.
(2) Figures to the right indicate marks.

1. Answer the following questions **(15 M)**

(a) Choose the best choice for the following questions: **(5 M)**

(i) If f_1 and f_2 are two functions from **R** to **R** such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$, then $(f_1.f_2)(x)$ is given by
(a) $x^3 + x^4$ (b) $x^3 - x^4$
(c) $x^4 + x^3$ (d) None of these

(ii) Which of the following is true about the poset $(Z^+, |)$?
(a) Zero is the least element (b) One is the least element
(c) There is no least element (d) None of these

(iii) A class contains 10 students with 6 men and 4 women. Number of ways to select a 4-member committee with 2 men and 2 women is given by
(a) 60 (b) 70 (c) 80 (d) 90

(iv) Suppose a bookcase shelf has 5 History texts, 3 Sociology texts, 6 Anthropology texts, and 4 Psychology texts. Number of ways a student can choose one text of each type is given by
(a) 360 (b) 460 (c) 560 (d) 660

(v) A loop is an edge connecting
(a) a vertex with itself (b) two distinct vertices
(c) no vertices (d) three distinct vertices

(b) Fill in the blanks for the following questions: **(5M)**

(i) A function f is said to be strictly _____________ if f(x) > f(y) for any x and y in the domain of f.

(ii) A relation R on a set A is called ___________if (a, a) ∈ **R** for every element a ∈ A.

(iii) The Gödel number of a word $w = a_3a_2a_1a_3a_4$ is_________

(iv) Suppose that a procedure can be broken down into two tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are _______ ways to do the procedure.

(v) Let G be a directed graph and v be a vertex of G. The number of edges beginning at v is called ___________.

(c) Answer the following questions: **(5M)**

(i) Why is f, defined by f (x) = 1/(x+1), not a function from **R** to **R**?

(ii) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} + 2$ for n = 1, 2, 3, . . . and suppose that $a_0 = 2$. What are a_1 and a_2?

TURN OVER

(iii) State inclusion exclusion principle.
(iv) Define a regular grammar.
(v) Define a directed graph.

2. Answer any *three* of the following: (15 M)

(a) Find the domain and range of the following functions:
i) the function that assigns to each nonnegative integer its last digit.
ii) the function that assigns to a bit string the number of bits in the string.

(b) Determine whether the function f from R to R defined by $f(x) = x - 2$ is bijective.

(c) Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation? Justify your answer.

(d) Define Lattice. Determine whether the posets ({1, 2, 3, 4, 5}, |) and ({1, 2, 4, 8, 16}, |) are lattices.

(e) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

(f) Define Fibonacci numbers. Formulate a recurrence relation for Fibonacci numbers.

3. Answer any *three* of the following: (15 M)

(a) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?

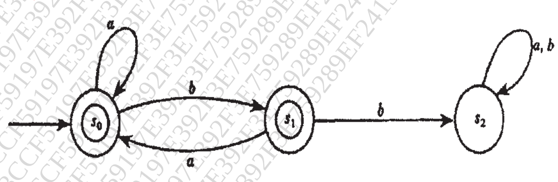
(b) State and prove Vandermonde's identity.

(c) State Pigeonhole principle. From any set of 13 integers, prove that there will be at least one pair which leaves the same remainder when divisible by 12.

(d) How many integers between 1 and 600 (both inclusive) are not divisible by both 3 and 5?

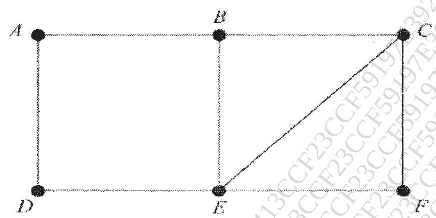
(e) Define a language L over an alphabet A. Let A= {a, b, c}. Find L^* where language $L = \{b^2\}$.

(f) Determine whether or not the automaton *M* in the following figure accepts the words: $w_1 = ababba$; $w_2 = baab$; $w_3 = \lambda$ the empty word.

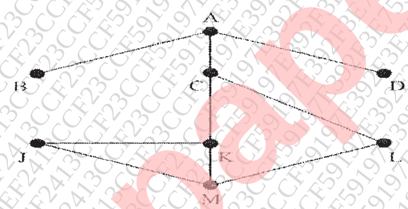


4. Answer any *three* of the following: (15 M)

(a) Consider the graph *G* in the following. Find (i) diam*(G)*, the diameter of *G* , (ii) *d(A,F)*, the distance from *A* to *F*.



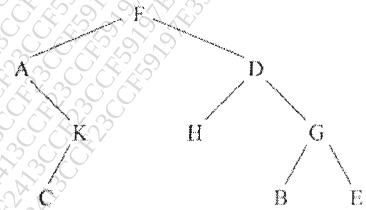
(b) Consider the graph G in the following figure (where the vertices are ordered alphabetically). (i) Find the adjacency structure of G. (ii) Find the order in which the vertices of G are processed using a depth-first search algorithm beginning at vertex A.



(c) Draw the graph G corresponding to each adjacency matrix:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

(d) Consider the binary tree T in the following figure. (i) Find the depth d of T. (ii) Traverse T using the post-order algorithm.



(e) Suppose a graph G contains two distinct paths from a vertex u to a vertex v. Show that G has a cycle.

(f) Consider the binary tree T in the following figure. Describe the tree T after (i) the node M and (ii) the node D are deleted.

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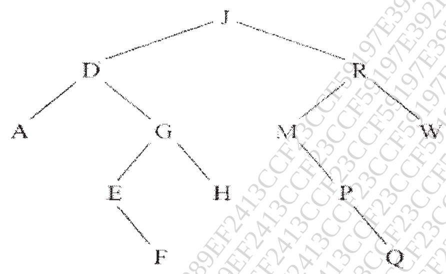

5. **Answer any *three* of the following:** **(15 M)**

(a) Draw the Hasse diagram for divisibility on the set {1, 2, 3, 5, 7, 11, 13}.

(b) How many solutions does the equation $x+y+z=11$ have, where x, y and z are non-negative integers with $x \leq 3$, $y \leq 4$ and $z \leq 6$?

(c) Draw all possible non similar binary trees *T* with four external nodes.

(d) Show that $a_n = n.2^n$ is a solution of the non-homogeneous linear recurrence relation $a_n = 2a_{n-1} + 2^n$.

(e) What is the language generated by phase structure grammar G?