

- N.B. 1) All questions are compulsory.
 2) Figures to the right indicate marks.
 3) Illustrations, in-depth answers and diagrams will be appreciated.
 4) Mixing of sub-questions is not allowed.

Q. 1 Attempt the following questions**(15M)**

- (a) Choose the best choice for the following questions
- (5M)**

- i) The absolute value of $3 + 4i$ is:
 a) 4 b) 5 c) 6 d) zero
- ii) In GF(2) field, $1 + 1$ is equal to
 a) 1 b) 0 c) both a) and b) d) none of these
- iii) How to declare the complex number in Python?
 a) (3, 4) b) Complex(3, 4) c) Complex (3, 4i) d) None of these
- iv) If a matrix is $R \times C$ and a vector is a C vector then the product is called
 a) Matrix-Matrix b) Vector-Matrix
 c) Vector-Vector d) Matrix-Vector
- v) Suppose $t = (1, 2, 4, 3)$, which of the following is incorrect?
 a) `print(t[3])` b) `t[3] = 45`
 c) `print(max(t))` d) `print(len(t))`

- (b) Fill in the blanks for the following questions
- (5M)**

- i) Any complex number multiplying by i , rotate it by _____
 ii) Set of all linear combinations of vectors is called _____
 iii) A rectangular array of m rows and n columns is called a _____
 iv) Norm of Vector $(1, 2, 3)$ is _____
 v) Every Subset of a linearly independent set is linearly _____

- (c) Answer the following questions
- (5M)**

- i) Solve: $1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 1$
 ii) Find dot product of $(1, 2)$, $(3, 4)$
 iii) Show with example matrix representation in python
 iv) Define the term Basis
 v) Define the term Inner Product Space

Q. 2 Attempt the following (Any THREE)(Each of 5Marks)**(15M)**

- (a) Find the square root of complex number $-5 + 12i$
 (b) Show that vectors $v_1 = (1, 0, 1)$, $v_2 = (2, 1, 4)$ and $v_3 = (1, 1, 3)$ do not span vector space.
 (c) Write a Python program to rotate a complex no by 90° , 180° and 270°

- (d) Check whether the vectors are linearly dependent
 $v_1=(1, -2, 1)$, $v_2=(2, 1, -2)$ and $v_3=(7, -4, 1)$.
- (e) Express $[(3 + 2i)/(2 + i)(1 - 3i)]$ in the form $x + iy$
- (f) Check whether the set of all pairs of real numbers of the form $(1, x)$ with operation $(1, y) + (1, y') = (1, y + y')$ and $k(1, y) = (1, ky)$ is a vector space.

Q. 3 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 4 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \quad c = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \quad D = [2 \ 4 \ 3 \ 1]$$

Compute the following if they exists.

- (a) A + B b) 3A c) B + 2D
- (b) Find the dimension of the vector space spanned by the vectors $(1, 1, -2, 0, -1)$, $(1, 2, 0, -4, 1)$, $(0, 1, 3, -3, 2)$, $(2, 3, 0, -2, 0)$ and also find the basis.
- (c) Check whether the set of functions are Linearly independent?
 $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$.
- (d) Explain Matrix-Vector and Vector-Matrix multiplication with example.
- (e) Write a python program to enter a matrix and check if it is invertible.
 if invertible exists then find inverse.
- (f) Show that vector $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ of R^3 form a basis of R^3

Q. 4 Attempt the following (Any THREE) (Each of 5Marks) (15M)

- (a) If $u = (2, 3, -1)$ and $v = (6, -3, -2)$
 Find a) $d(u, v)$ b) $u - v$ c) $2u + 3v$
- (b) Verify Pythagorean Theorem for $u = (1, 0, 2, -4)$ and $v = (0, 3, 4, 2)$
- (c) If $x, y, z \geq 0$
 Show that $(x^2 + y^2 + z^2)^{1/2} \geq (1/13)(3x + 4y + 12z)$
- (d) Find inner product, angle, orthogonality for
 $P = -5 + 2x - x^2$, $q = 2 + 3x^2$
- (e) Find the vector orthogonal to both $u = (-6, 4, 2)$ and $v = (3, 1, 5)$
- (f) Write a python program to find orthogonal projection u on v .

Q. 5 Attempt the following (Any THREE) (Each of 5Marks) (15M)

- (a) Express the following as a linear combination of $v_1=(-2, 1, 3)$, $v_2=(3, 1, -1)$ and $v_3=(-1, -2, 1)$ with $w=(6, -2, 5)$

- (b) Write a python program to convert a 2×2 matrix to row echelon form
(c) Verify Cauchy's Schwartz's inequality $u = (1, 2, -1)$ and $v = (3, 2, -1)$
(d) Find eigen Values and eigen vectors of

$$A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

- (e) Construct an orthonormal basis of R^2 by Gram Schmitt Process
 $S = \{(3, 1), (4, 2)\}$
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