**Q.P. Code : 24510**

**Total Marks: 80** **Time Duration: 3Hr**

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Figures to the right indicate full marks.

**Maximum Marks**

**Q1. a)** Find the Laplace transform of $\frac{1}{t} e^{-t} \sin t$. [5]

**b)** Find the inverse Laplace transform of $\frac{1}{\sqrt{2s+1}}$. [5]

**c)** Show that the function $f(z) = \sinh z$ is analytic and find $f'(z)$ in terms of z. [5]

**d)** Find the Fourier series for $f(x) = x$ in $(0, 2\pi)$. [5]

**Q2. a)** Use Laplace transform to prove $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$. [6]

**b)** If $\{f(k)\} = \begin{cases} 4^k, k < 0 \\ 3^k, k \geq 0 \end{cases}$, find $Z\{f(k)\}$. [6]

**c)** Show that the function $u = \cos x \cosh y$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function. [8]

**Q3. a)** Find the equation of the line of regression of Y on X for the following data. [6]

| X | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| Y | 11 | 14 | 14 | 15 | 12 | 17 | 16 |

**b)** Find the bilinear transformation which maps the points 1, -i, 2 on z-plane onto 0, 2, -i respectively of w-plane. [6]

**c)** Find half range sine series for $f(x) = \begin{cases} x & , 0 < x < \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} < x < \pi \end{cases}$, Hence find the sum of $\sum_{(2n-1)}^{\infty} \frac{1}{n^4}$. [8]

**Q4. a)** Find the inverse Laplace transform by using convolution theorem $\frac{1}{(s-a)(s+a)^2}$. [6]

**b)** Calculate the coefficient of correlation between X and Y from the following data. [6]

| X | 8 | 8 | 7 | 5 | 6 | 2 |
|---|---|---|---|---|---|---|
| Y | 3 | 4 | 10 | 13 | 22 | 8 |

**c)** Find the inverse Z-transform of [8]

i) $\frac{1}{(z-a)^2}$ $|z| < a$

ii) $\frac{1}{(z-3)(z-2)}$ $|z| > 3$

**Q5.a)** Using Laplace transform evaluate $\int_0^{\infty} e^{-t}\left(1+2t-t^2+t^3\right)\, H(t-1)dt$ . [6]

**b)** Show that set of functions $\cos x, \cos 2x, \cos 3x$ .... Is a set of orthogonal functions over $[-\pi , \pi]$. Hence construct a set of orthonormal functions. [6]

**c)** Solve using Laplace transform [8]

$(D^3 - 2D^2 + 5D)y = 0$, with $y(0) = 0, y'(0) = 0, y''(0) = 1$.

**Q6.a)** Find the complex form of Fourier series for $f(x) = 2x$ in $(0 , 2\pi)$. [6]

**b)** If $f(z) and \overline{f(z)}$ are both analytic, prove that $f(z)$is constant. [6]

**c)** Fit a curve of the form $y = ab^x$ to the following data. [8]

| X | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Y | 151 | 100 | 61 | 50 | 20 | 8 |

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