

ELEC (CBCS) AM-III

Q.P. Code: 24961

23/11/17

Duration : 3 Hours

Max. Marks :80

N.B. 1) Question No. 1 is compulsory .

2) Attempt any three questions out of the remaining five questions.

3) Figures to the right indicate full marks

1. (a) Find  $L\left\{\frac{d}{dt}\left(\frac{\sin 4t}{t}\right)\right\}$  5
- (b) Find the Fourier series for  $f(x) = 5x^2$  in  $(-\pi, \pi)$  5
2. (c) Show that the vector  $\vec{F} = \frac{-yi + xj}{x^2 + y^2}$  is irrotational. 5
- (d) Determine constant a,b,c,d if  
 $f(z) = (10x^2 + ay^2 + 8bxy) + i(5cxy + 7dx^2 + 2x^2)$  is analytic. 5
2. (a) Prove that  $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$  6
- (b) Solve using Laplace transform  $(D^3 - 3D - 2)y = 590.t^3.e^{3t}$   
 $y(0) = 0, y'(0) = 0, y''(0) = 0$ . 6
- (c) Find half range cosine series for  $\cos \alpha x$  in  $(0, \pi)$ , where  $\alpha$  is  
 not an integer and hence show that  $\sum_{n=1}^{\infty} \frac{1}{\alpha^2 - n^2} = \frac{\alpha \pi \cot \alpha \pi}{1 + 2\alpha^2}$  8
3. (a) If  $u = (x^2 + y^2 + z^2)$  Prove that  $\text{Curl}(\text{grad } u) = \vec{0}$ . 6
- (b) Find Fourier for  $f(x) = \begin{cases} x & \text{odd} \\ x^2 & \text{even} \end{cases}$  in  $(0, 2\pi)$ . 6
- (c) Evaluate  $\int_0^{\infty} e^{-4t} \int_0^t (u \sinh^2 u)^2 \cdot \cosh 5u e^{3u} du dt$  8
4. (a) Find the bilinear transformation which maps the points  
 $z = 1, i, -1$  onto the points  $w = i, 0, -i$  6
- (b) By using Stoke's theorem evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  
 $\vec{F} = (8x + 2y)i - 4z^2j - y^2zk$  and C is the boundary of the  
 hemisphere  $x^2 + y^2 + z^2 = a^2, z = 0$ . 6
- (c) Find Inverse Laplace transform  
 i)  $\left\{\frac{5s+3}{s^2+6s+25}\right\}$  ii)  $\log\left\{\frac{s^2+64}{s^2+16}\right\}$  8

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5. (a) Define Orthogonal set of functions on (a,b), Show that the functions  $f_1(x) = 1$ ,  $f_2(x) = 3x$  are orthogonal on (-2,2). Determine the constants P, Q such that  $f_3(x) = Px^2 + Qx + 9$  is orthogonal to both  $f_1(x)$  &  $f_2(x)$  on the same interval. 6

(b) Find the analytic function  $f(z) = u + iv$  in terms of Z if  $3u - 7v = x^3 + x^2 - 3xy^2 - y^2 - 3yx^2 + y^3 - 2xy$ . 6

(c) Verify Green's theorem for  $\int_C (4xy - x^2)dx + (2x + 6y^2)dy$ , C is the closed curve in the XY-plane bounded by  $y = x^2$  and  $x = y^2$ . 8

6. (a) Find Laplace transform of  $f(x) = \begin{cases} \sin 7t & 0 < t < \pi/2 \\ 2 & \pi/2 < t < \pi \end{cases}$  and  $f(t) = f(t + \pi)$ . 6

(b) Find the invariant points of the Bilinear transformation  $w = \left(\frac{4z-9}{z-2}\right)$ , also express it in the normal form. 6

(c) Obtain Complex form of Fourier series for  $f(x) = \sinh x$  in  $(-l, l)$  8

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