

SE Electrical Engg. Choice Base 08/05/2018 1/2  
 App. Maths - III

Q.P. Code: 24962

Duration: 3 Hours

Max. Marks: 80

- Note: 1. Questions No. 1 is compulsory.  
 2. Attempt any 3 Questions from the remaining questions.  
 3. Figures to the right indicate carries full marks.

- Que. 1 a. Find Laplace transform of  $e^{-t}t\cos 2t \cdot \sin 4t$  5  
 b. Find Fourier expansion for  $f(x) = |x|$  in  $(-\pi, \pi)$  5  
 c. Prove that  $\vec{F} = \frac{\vec{r}}{r^3}$  is solenoidal. 5  
 d. If  $f(z) = (r^2 \cos 2\theta + ir^2 \sin p\theta)$  is analytic find p. 5
- Que. 2 a. Prove that  $u = e^{2x} \cos 2y$  is harmonic function, hence find it's corresponding harmonic conjugate orthogonal 6  
 b. By using convolution theorem, find the inverse Laplace Transform of  $\frac{s}{(s^2+4)(s^2+9)}$  6  
 c. Find Fourier series for  $f(x) = x \sin x$  in  $(0, 2\pi)$  8
- Que. 3 a. Prove that a vector field  $\vec{F}$  is given by  $\vec{F} = (y \sin z - \sin x)i + (x \sin z + 2yz)j + (xy \cos z + y^2)k$  is irrotational, hence find its scalar potential. 6  
 b. Find analytic function  $f(z)$ , whose real part is  $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$  6  
 c. By using Laplace transform, solve  $y'' + 2y' + 5y = e^{-t} \sin t, y(0) = 0, y'(0) = 1$  8
- Que. 4 a. Prove that  $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$  6  
 b. Evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (2x - y)i - yz^2j - y^2zk$ , where C is the boundary of the surface of hemisphere  $x^2 + y^2 + z^2 = a^2$  lying above the xy-plane. 6  
 c. Find inverse Laplace Transform of i.  $\frac{(s+1)e^{-\pi s}}{s^2+2s+5}$  ii.  $\frac{1}{s} \log\left(\frac{s+2}{s+1}\right)$  8

Que 5 a. Show that the functions  $\{ \sin x, \sin 3x, \sin 5x, \dots \}$  are orthogonal

in  $[0, \frac{\pi}{2}]$  and find the corresponding set of orthonormal functions. 6

b. Show that under the transformation  $w = \frac{1}{z}$  the circle  $(x-3)^2 + y^2 = 2$  the circle is mapped to the circle  $(u - \frac{3}{7})^2 + v^2 = \frac{2}{49}$  6

c. Verify Green's Theorem in the plane for  $\oint (x^2 - y)dx + (2y^2 + x)dy$  around the boundary of the region defined by  $y = x^2$  and  $y = x$ . 8

Que 6 a. By using Laplace transform, evaluate  $\int_0^{\infty} e^t \frac{\sin^2 t}{t} dt$  6

b. Find a bilinear transformation which maps  $z=1, i, -1$  into  $w=0, i, -\infty$  and hence find the fixed points 6

c. Find the Fourier integral representation of  $f(x) = \begin{cases} e^{ax}, & x \leq 0 \\ e^{-ax}, & x \geq 0 \end{cases}$  8

and hence S.T.  $\int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2a} e^{-ax}; x > 0, a > 0$