Q. P. Code: 24393

(3 Hours) **[Total Marks: 80]**

N.B.: 1) Question No. 1 is Compulsory.

2) Answer any **THREE** questions from **Q.2 to Q.6**.

3) Figures to the right indicate full marks.

Q 1. a) Evaluate the Laplace transform of $\sqrt{1+\sin t}$ [5]

b) Find directional derivative of $\phi = 4xz^2 + x^2yz$, at $(1,-2,-1)$ in direction of $2i - j - 2k$ [5]

c) Find orthogonal trajectories of the family of curves $e^x \cos y - xy = c$.[5]

d) Obtain half range sine series for $f(x)=x, \quad 0<x<2$. [5]

Q 2. a) If $u+v = e^{2x}(x\cos 2y - y\sin 2y)$ then find analytic function $f(z)$ by Milne Thomson Method [6]

b) Find the Fourier series for $f(x) = 9 - x^2$, $-3 \le x \le 3$ [6]

c) Find the Laplace transform of the following

i) $L[t\sqrt{1+\sin t}]$ ii) $L\left[\frac{\sinh 2t}{t}\right]$ [8]

Q 3. a) Using Convolution theorem, find Inverse Laplace of $\frac{s}{(s^2+4)^2}$. [6]

b) Prove that $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \frac{(3-x^2)}{x^2}\cos x\right]$. [6]

c) Find Fourier series for $f(x) = (\pi - x)^2$ in $0 \le x \le 2\pi$. Hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$ [8]

Q 4 a) Find the Fourier transform of $f(t) = e^{-|t|}$ [6]

b) Show that the function $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on $(-1,1)$ and determine the constant A & B so that functions $f_3(x) = 1 + Ax + Bx^2$ is orthogonal to both $f_1(x)$ and $f_2(x)$ on that interval. [6]

c) Find bilinear transformation which maps the points z=1, i,-1 onto the points w=i, 0,-i hence

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find the image of $|z|<1$ on to w plane find invariant points of this transformation [8]

Q 5 a) Solve using Laplace Transform $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=te^{-t}$ given $y(0)=4$ and $y'(0)=2$. [6]

b)Find Complex form of the Fourier series for $f(x)=e^{ax}$ in $-\pi<x<\pi$ where 'a' is a real constant. Hence deduce that $\frac{\pi}{a\sinh a\pi}=\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{n^2+a^2}$ [6]

c) Verify Green's Theorem in the plane for $\oint_C (3x^2-8y^2)\,dx+(4y-6xy)dy$ where C is the boundary of the region defined by $y = x^2$ and $y = \sqrt{x}$. [8]

Q 6. a) Prove that $J_n''(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$ [6]

b) Find the map of the line x-y=1 by transformation $w=\frac{1}{z}$ [6]

c) Evaluate $\iint_S \bar{F}.d\bar{s}$ where $\bar{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ where S is the region bounded by $x^2+y^2=4$, $z=0$, $z=3$ using Gauss divergence theorem. [8]

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