

(3hours)

[Total marks: 80]

- N.B.** 1) Question No. 1 is compulsory.  
2) Answer **any Three** from remaining  
3) Figures to the right indicate full marks



1. a) Find Laplace transform of  $f(t) = \int_0^t u e^{-3u} \sin u du$ . 5  
b) Show that the set of functions  $\{\cos nx, n = 1, 2, 3 \dots\}$  is orthogonal on  $(0, 2\pi)$ . 5  
c) Does there exist an analytic function whose real part is  $u = k(1 + \cos \theta)$ ? Give justification. 5  
d) The equations of lines of regression are  $x + 6y = 6$  and  $3x + 2y = 10$ . Find  
i) means of  $x$  and  $y$ , ii) coefficient of correlation between  $x$  and  $y$ . 5
2. a) Evaluate  $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$ . 6  
b) Find the image of the triangle bounded by lines  $x = 0, y = 0, x + y = 1$  in the  $z$ -plane under the transformation  $w = e^{i\pi/4} z$ . 6  
c) Obtain Fourier series of  $f(x) = x^2$  in  $(0, 2\pi)$ . Hence, deduce that – 8  
$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$
3. a) Find the inverse Laplace transform of  $F(s) = \frac{s}{(s^2+4)^2}$ . 6  
b) Solve  $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$ , with  $u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1 - x)$   
taking  $h = 0.1$  for three time steps up to  $t = 1.5$  by Bender –Schmidt method. 6  
c) Using Residue theorem, evaluate

i)  $\int_0^{2\pi} \frac{d\theta}{5 - 4\cos \theta}$

ii)  $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2}$  8

[TURN OVER]

4. a) Solve by Crank –Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ,

$$u(0, t) = 0, u(5, t) = 100, u(x, 0) = 20 \text{ taking } h = 1 \text{ for one-time step. } 6$$

b) Obtain the Taylor's and Laurent series which represent the function

$$f(z) = \frac{z-1}{z^2-2z-3} \text{ in the regions, i) } |z| < 1 \text{ ii) } 1 < |z| < 3 \quad 6$$

c) Solve  $(D^2 + 4D + 8)y = 1$  with  $y(0) = 0$  and  $y'(0) = 1$  where  $D \equiv \frac{d}{dt}$  8

5. a) Find an analytic function  $f(z) = u + iv$ , if 6

$$u = e^{-x}\{(x^2 - y^2) \cos y + 2xy \sin y\}$$

b) Find the Laplace transform of

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases} \text{ and } f(t+2) = f(t) \text{ for } t > 0. \quad 6$$

c) Obtain half range Fourier cosine series of  $f(x) = x, 0 < x < 2$ . Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

6. a) If  $f(a) = \int_C \frac{4z^2+z+4}{z-a} dz$  where  $C$  is the ellipse  $4x^2 + 9y^2 = 36$ .

Find, i)  $f(4)$  ii)  $f'(-1)$  and iii)  $f''(-i)$  6

b) Use least square regression to fit a straight line to the following data, 6

<b>x</b>	5	10	15	20	25	30	35	40	45	50
<b>y</b>	17	24	31	33	37	37	40	40	42	41

c) A string is stretched and fastened to two points distance  $l$  apart. Motion is started by displacing the string in form  $y = a \sin(\pi x / l)$  from which it is released at a time  $t = 0$ . If the vibrations of a string is given by  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ , show that the displacement of a point at a distance  $x$  from one end at time  $t$  is given by  $y(x, t) = a \sin(\pi x / l) \cos(\pi ct / l)$  . 8