

(3 hours)

Max. Marks: 80



- N.B. (1) Question No. 1 is compulsory.
 (2) Answer any three questions from Q.2 to Q.6.
 (3) Use of Statistical Tables permitted.
 (4) Figures to the right indicate full marks.

Q.1 (a) Find all the basic solutions to the following problem:

$$\text{Maximise } z = x_1 + 3x_2 + 3x_3$$

$$\text{subject to } x_1 + 2x_2 + 3x_3 = 4$$

$$2x_1 + 3x_2 + 5x_3 = 7$$

$$x_1, x_2, x_3 \geq 0$$

05

(b) Evaluate $\int_0^{1+2i} z^2 dz$, along the curve $2x^2 = y$.

05

(c) A random sample of size 16 from a normal population showed a mean of 103.75 cm & sum of squares of deviations from the mean 843.75 cm^2 . Can we say that the population has a mean of 108.75?

05

(d) If $A = \begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$, find $\sin A$

05

Q.2 (a) Evaluate $\int_c \frac{dz}{z^3(z+4)}$, where c is the circle $|z| = 2$

06

(b) Memory capacity of 9 students was tested before & after a course of mediation for a month. State whether the course was effective or not from the data below

06

Before	10	15	9	3	7	12	16	17	4
After	12	17	8	5	6	11	18	20	3

(c) Solve the following LPP using Simplex Method

$$\text{Maximise } z = 6x_1 - 2x_2 + 3x_3$$

$$\text{subject to } 2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 + 4x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

08

Q.3 (a) Find the Eigen values and Eigen vectors of the following matrix.

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

06

(b) For a normal distribution 30% items are below 45% & 8% are above 64. Find the mean & variance of the normal distribution.

06

(c) Obtain Laurent's series for $f(z) = \frac{1}{z(z+2)(z+1)}$ about $z = -2$

08

[Turn over

Q.4 (a) An ambulance service claims that it takes on an average 8.9 min to reach the destination in emergency calls. To check this the Licensing Agency has then timed on 50 emergency calls, getting a mean of 9.3 min with a S.D. 1.6 min. Is the claim acceptable at 5% LOS? 06

(b) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$ 06

(c) (i) If 10% of the rivets produced by a machine are defective, find the probability that out of 5 randomly chosen rivets at the most two will be defective. 04+04
 (ii) If x denotes the outcome when a fair die is tossed, find M.G.F. of x and hence, find the mean and variance of x .

Q.5 (a) Check whether the following matrix is Derogatory or Non-Derogatory:

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
 06

(b) Justify, if there is any relationship between sex and color for the following data.

Color	Male	Female
Red	10	40
White	70	30
Green	30	20

06

(c) Use the dual simplex method to solve the following L.P.P.

Minimise $z = 2x_1 + x_2$
 subject to $3x_1 + x_2 \geq 3$
 $4x_1 + 3x_2 \geq 6$ 08
 $x_1 + 2x_2 \leq 3$
 $x_1, x_2 \geq 0$

Q.6 (a) Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1} .

Where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 06

(b) The Probability Distribution of a random variable X is given by

X	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	$2k$	0.3	k

06

Find k , mean and variance.

(c) Using Kuhn-Tucker conditions, solve the following NLPP

Maximise $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$
 subject to $2x_1 + 5x_2 \leq 98$ 08
 $x_1, x_2 \geq 0$